Differential Equations: • Relate derivative of a function to the function itself

• Solution is a <u>function</u>

Ex: $y' = ky$ (k constant)

$\rightarrow y(t) = Ce^{kt}$

C ← can be any #

$5e^{kt}, \; e^{kt}, \; -e^{kt}$

Ex: $y' = \underbrace{ay+b}$

Linear function of y. "Linear differential equations"

Ex: $v' = \underbrace{g}_{\text{gravity}} - \underbrace{v^2}_{\text{air friction}}$ object falling

$y' = -\underline{\sin(y)}$ pendulum

Qualitative analysis: Can't explicitly solve. Can you extract useful info?

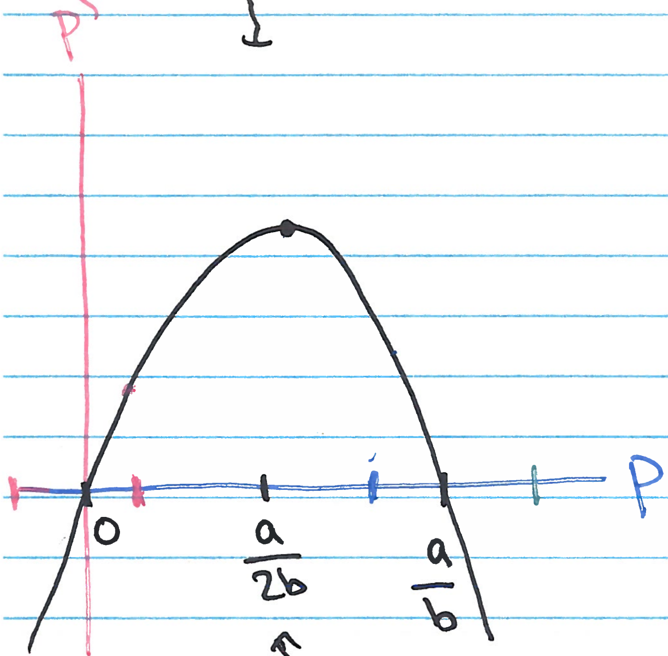
$P' = \underbrace{aP}_{\text{birth}} - \underbrace{bP^2}_{\text{death}}$

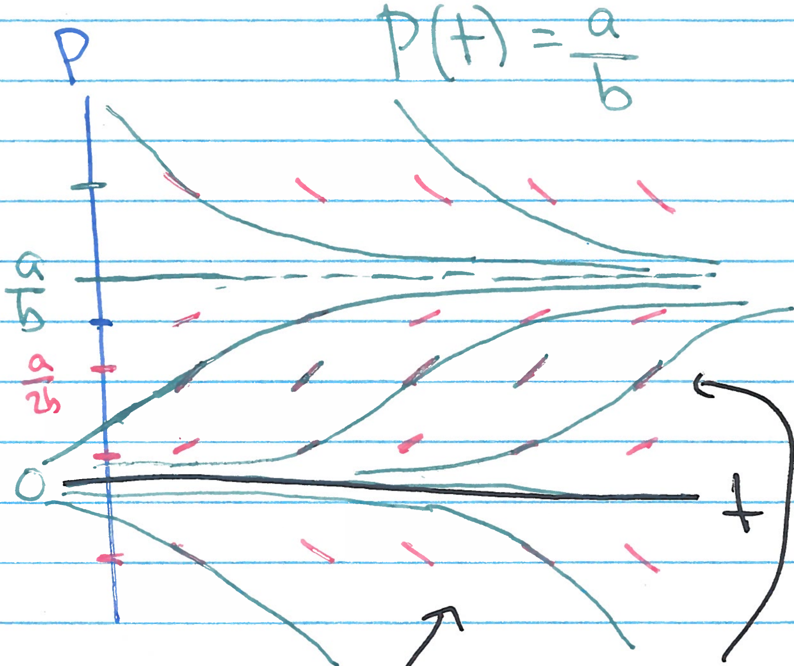
Birth rate per capita $= a$

Death rate per capita $= bP$

Logistic Equation

$$\underbrace{P' = aP - bP^2}_{} = bP\left(\frac{a}{b} - P\right) \rightsquigarrow P(t) = 0,\ P(t) = \frac{a}{b}$$





Height here = Slope here

"Slope field":

it tells you the direction at every point of trajectories

Q: Given that P' is a function of P, we know

If $P(t)$ is a solution then

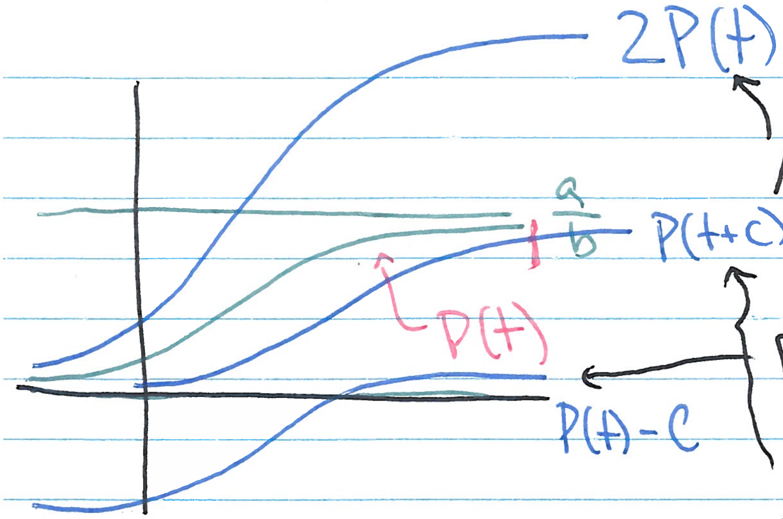
A) So is $C \cdot P(t)$

B) So is $P(t) + C$

C) So is $P(t+C)$

D) Confused, help!

$$\boxed{\begin{array}{l} P = \frac{a}{b} \text{ is stable} \\ P = 0 \text{ is unstable} \end{array}}$$



A) Can't happen b/c of slope fields X

B) Again, slope fields X

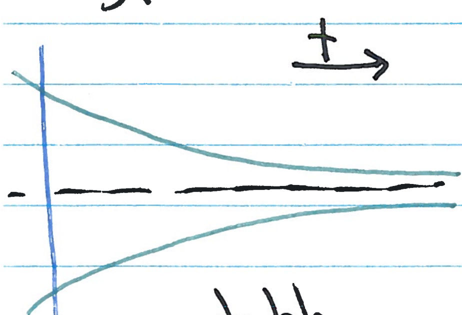
C) Is a solution

~~All soln~~

You can horizontally traslate any solution & get more solutions

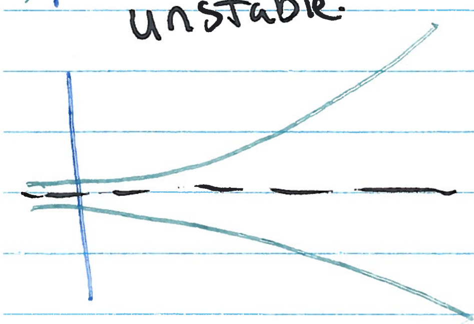
A steady state is **stable** if nearby trajectories go toward it

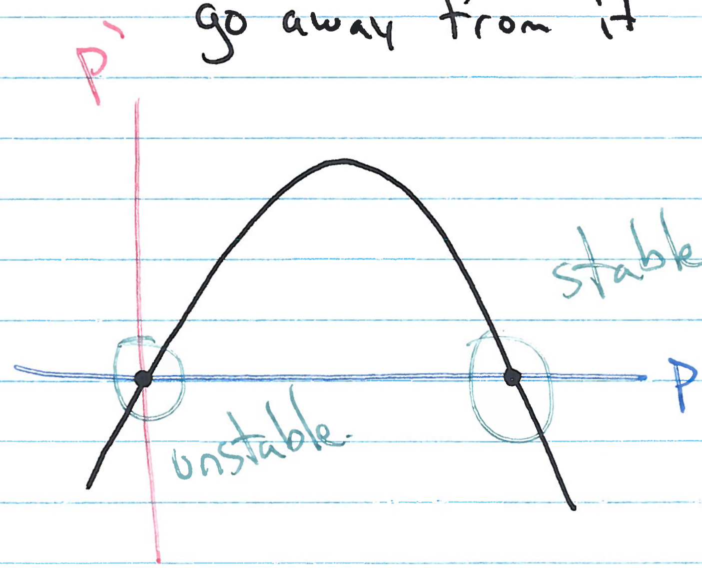
stable



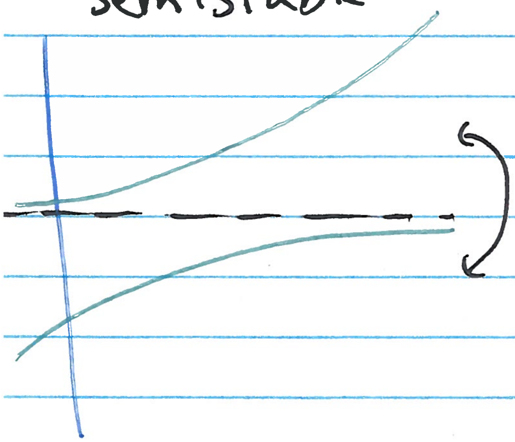
unstable if nearby trajectories go away from it

unstable

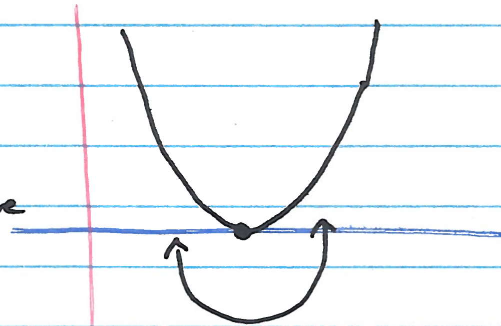




semistable



slope positive above & below



height positive above & below